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Instant Chaos, Extreme Parametric Uncertainty, and Sustained Chaotic Transients in Nonlinear Continuum Mechanics

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Instant chaos, extreme parametric uncertainty, and sustained chaotic transients in nonlinear continuum mechanics

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ABSTRACT

Bifurcation to high-dimensional hyperchaos is observed in a driven coupled pendulum-flexible rod system. When the rod is in resonance with the pendulum, the system changes from a low dimensional attractor to a high dimensional attractor abruptly. It is shown that high dimensional chaotic dynamics is hysteretic, and exhibits extreme sensitivity with respect to small parameter changes. Such sensitivity poses a problem in obstructing predictability in mechanics. A brief discussion of sustaining chaos to prevent resonance is included.

INTRODUCTION

One of the simplest problems studied in nonlinear physics is that of the forced, damped pendulum (Baker, 1996), (Miles, 1993). Studied extensively in isolation, the pendulum is always attached to some support, such as a stiff rod, and is used in many active control mechanisms, such as vibration absorbers (Starrett,1995), (Cuvalci, 1996). If the rod is sufficiently stiff, one expects the pendulum dynamics to be slightly perturbed from the ideal infinitely stiff case. We review some new dynamical behavior in the numerical simulations of a forced, damped pendulum coupled to a linear rod which is flexible. We examine the system when it is operating in a resonant mode, where the pendulum frequency is half that of the fundamental frequency of the rod, as well as a non-resonant case. It is known that when the rod is sufficiently stiff, the dynamics resides on a global slow invariant manifold; i.e., the rod is slaved to the motion of the pendulum (Georgiou and Schwartz, 1999). As a result of this slaving motion, the dynamics is a perturbation of a parametrically driven pendulum.

However, when operating at resonance, there exists some critical amplitude of the driving force that causes a discontinuous change from periodic behavior to hyper-chaotic behavior, where there are two or more positive Lyapunov exponents.

In contrast, non-resonant dynamics exhibits multiple chaotic attractors, one which is statistically constrained to a low dimensional surface, and one which is not. In both cases, multiple attractors as well as long lived chaotic transients exist. Such multiplicity gives rise to an obstruction in predicting the type of asymptotic behavior due to small uncertainties in parameters and/or data (Schwartz, et al, 1999). Either chaotic or periodic behavior may result for given initial data. To eliminate the possibility of periodic resonant behavior, the results of a new parametric control procedure to sustain chaos are presented (Schwartz and Triandaf, 1996), (Triandaf and Schwartz, 2000).

MODEL EQUATIONS

The dynamical system we consider (see Fig.1) consists of a planar pendulum of length L_p, mass M_p, and viscous dissipation coefficient C_p attached to the lower end B of a linear viscoelastic rod of length L_r, cross-section A_r, mass density ρ_r, elasticity modulus E_r, coefficient of internal viscous dissipation C_r.

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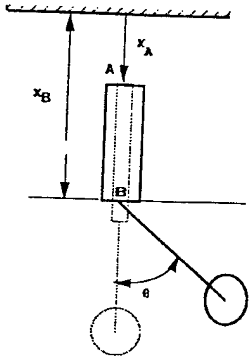

Figure 1. Rod-Pendulum Geometry.

The upper end A of the rod is subject to a periodic motion $x_A(t)$. Let $u(x, t)$ and $\theta(t)$ denote respectively the displacement field of the rod with respect to A and the angular displacement of the pendulum mass.

Let $\xi = \frac{x}{L}$, $\tau = \omega_p t$ (derivative with respect to τ is denoted by a "dot", where $\omega_p^2 = \frac{g}{L}$ is the uncoupled pendulum natural frequency denote normalized spatial and temporal variables. Letting $U = \frac{u}{L}$ denote the dimensionless displacement field for the rod, the dynamics of the rod-pendulum configuration are described by:

$$\ddot{\theta} = -[1 - \ddot{V}_B(\tau) + \ddot{X}_A(\tau)]\sin\theta - 2\zeta_p\dot{\theta}$$

$$\frac{\mu^2\pi^2}{4}V_{,\tau\tau}(\xi, \tau) = V_{,\xi\xi}(\xi, \tau) + 2\zeta_r\mu V_{,\tau\xi\xi}(\xi, \tau) - \frac{\mu^2\pi^2}{4}\ddot{X}_A(\tau) \quad (1)$$

$$V(0, \tau) = 0$$

$$V_{,\xi}(1, \tau) = -\frac{\mu^2\beta\pi^2}{4}[1 - T(\theta, \dot{\theta}, \tau)]\cos\theta \quad (2)$$

$$T(\theta, \dot{\theta}, \tau) = \dot{\theta}^2 + [1 - \ddot{X}_B(\tau)]\cos\theta$$

The details of the model and parameter scalings are given in (Georgiou and Schwartz, 1999). The physical parameters that control the coupling between the pendulum and the rod are the frequency ratio $\mu = \frac{\omega_p}{\omega}$ and the mass ratio β. For fixed β, the limit of coupled system Eqs. (1-2) as $\mu \to 0$ describes the motions of the parametrically forced uncoupled pendulum. A modal expansion of the displacement field reduces the coupled system to the following set of N coupled oscillators. The phase space of our truncated system for analysis is given by the $2N + 3$ dimensional vector (Ψ, Z) where $Z = ((\{z_1, z_2\}, \{z_3, z_4\}) \ldots \{z_{2N-1}, z_{2N}\})$ and $\Psi = (\theta, \dot{\theta}, \psi_3)$. Here Z represents the scaled states of the first N rod modal oscillators, and ψ_3 represents the angle of the periodic forcing.

SPATIO-TEMPORAL BEHAVIOR

Resonance

We study the resonance situation where the rod has a fundamental natural frequency of 2, and the drive frequency i also equal to 2. (Recall the natural frequency of the pendulum unity, making $\mu = \frac{1}{2}$.) The other fixed parameters of the normalized model are the mass ratio (unity), and damping coefficients. The variable control parameter is forcing amplitude α. We plot in Fig. 2 a bifurcation diagram of the attractor of the modal expansion to Eqs. (1-2) that is observed for the angular velocity of the pendulum sampled at the period of the drive. The control parameter throughout the paper is the amplitude of periodic forcing, α.

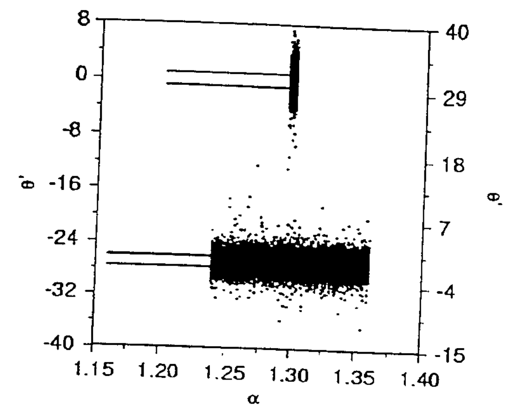

Figure 2. Bifurcation diagram of pendulum velocity sampled at the forcing frequency as a function of α. The coupling parameter is $\mu = \frac{1}{2}$.

The 2 to 1 internal resonance in the system results in a stable period 2 motion in the pendulum for a wide range of parameter values, including α very small. (The linear coupled modes exhibit period 1 behavior when the pendulum is in a period 2 cycle.) Notice that there appears to be no period doubling route to chaos as the parameter is increased. That is, the period 2 cycle appears to undergo instability directly to a chaotic attractor. Also, there is a region of hysteresis between the period 2 and the chaotic attractor.

When viewing the dynamics of extended behavior in spatially extended systems, in general it is not sufficient to represent the dynamics by sampling data at individual points, as in the method of time delay embedding (Schwartz and Triandaf, 1996b). instead, the method of snapshots (Sirovich, 1987) is a version of the Karhunen-Loeve method to analyze the data, and computes the topological dimension of the attractor.

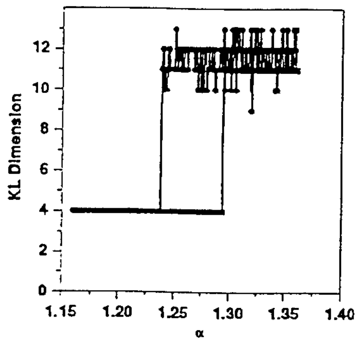

Figure 3. KL dimension as a function of α exhibiting hysteresis. Notice that when chaos appears in resonance, it is high dimensional.

Figure 3 shows the dimension as a function of the forcing amplitude. Plotted are two runs for both increasing and decreasing α. For low parameter values, the dimension is just 4. As the parameter is increased past the critical onset of chaos, we see a dramatic change in the size of the dimension, which is now more than three times that of the slaved periodic motion. The system must have excited more modes in order to generate such a high dimension.

Non-resonance

If the system is tuned away from resonance, the situation becomes more complicated. Whereas resonance had periodic oscillations residing on a low dimensional manifold, chaotic oscillations now appear for both small and large μ, which designates a weak and strong coupling, respectively. In Fig. 4, projection of the motion of the pendulum onto the $(\theta, \dot{\theta})$ plane shows the phase portrait when the dynamics is sampled at the forcing period. Notice the structure that appears due to the manifolds which bound the motion on a two-dimensional surface. In contrast, Fig. 5 shows the same projection when the coupling between the rod and the pendulum is large. Here the motion is not constrained by any topological structure since the dynamics is now higher dimensional.

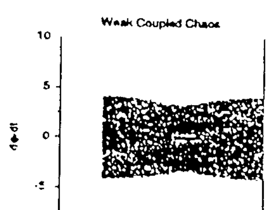

Figure 4. Low dimensional chaos, when $\mu = 0.025$.

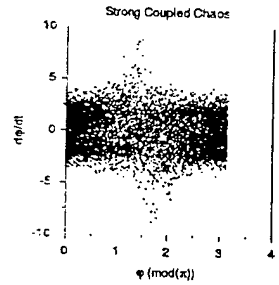

Figure 5. High dimensional chaos, when $\mu = 0.288$

Examination of the KL dimension reveals that although it is low for the weakly coupled case, it is not necessarily as low as the resonant case. This is due in part to intermittent escape off the manifolds. To see this, we explicitly compute the dynamics using Eqs. 1-2, and compare it to the dynamics approximated on the manifold using a singular (or center) manifold expansion (Carr, 1981), (Schwartz and Georgiou, 1999). The error between the two is then explicitly computed at the rod tip. For small coupling, the error is small for most of the time, but is not uniformly small, since the scale is absolute. The dynamics does in fact leave the manifold intermittently. Therefore, on average, the dynamics remains low dimensional. On the other hand, the strongly coupled case shows that the dynamics rarely visits the low dimensional manifold.

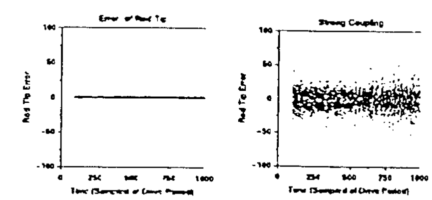

Figure 6. The error between the actual and predicted dynamics of the rod at the tip for weak coupling (left) and strong coupling (right).

EXTREME PARAMETRIC SENSITIVITY

The previous discussion should make it clear that for either the near-resonant case or non resonant case, there exist multiple attractors. When the attractors are chaotic and non-identical, one needs to classify the attractors. One way to achieve this is to compute the number of positive Lyapunov exponents for each attractor. This gives a statistical measure of the average number of unstable directions along the attractor. We have done this for the near-resonant case, and have found the existence of three distinct attractors: periodic, chaotic with one positive Lyapunov exponent, and one chaotic attractor with two positive Lyapunov exponents. (See (Schwartz, Wood, Georgiou, 1999) for computational details.) Figure 7 illustrates the complex intertwining in parameter space of the three attractors. Such complexity forces one to make large errors in the long time prediction of the type of behavior one expects. That is, for arbitrary small amounts of uncertainty in the parameter value, it is almost impossible to predict the outcome observed even statistically.

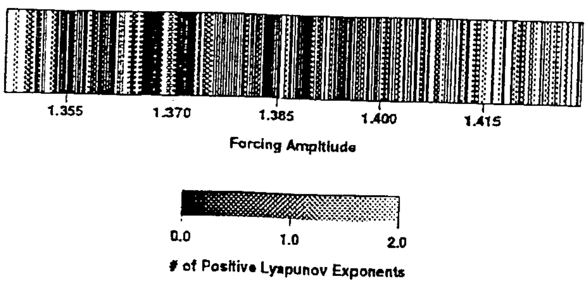

Figure 7. Number of positive Lyapunov exponents as a function of the forcing amplitude in the near resonance parameters.

We perform the following uncertainty experiment. Pick parameter α_0, and measure number of positive Lyapunov Exponents, $N_{LE}(\alpha_0)$. Perturb the parameter by a small amount, ε, called the uncertainty, which yields $\alpha_1 = \alpha_0 + \varepsilon$. If $N_{LE}(\alpha_1) \neq N_{LE}(\alpha_0)$, then α_0 'is defined to be *uncertain.* We then plot log of uncertain points versus the log of the uncertainty. The result is shown in Fig. 8. he slope is computed to be about 0.012, The dashed line designates what a good experiment would yield.

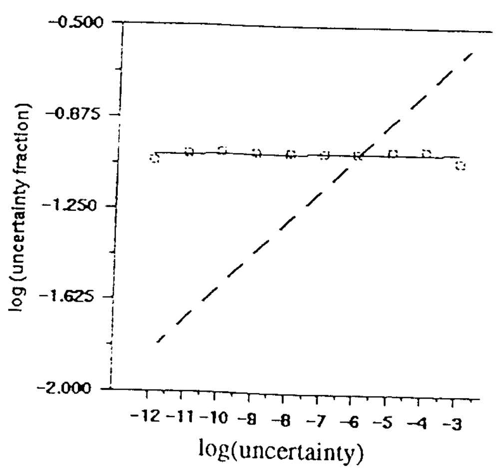

Figure 8. Fraction of uncertainty parameter values as a function of uncertainty. The dashed line illustrates an ideal experiment for prediction. In extreme parametric uncertainty, large increases in precision (decreases in uncertainty) do not significantly decrease the number of uncertain points. Hence, the obstruction to predictability.

DISCUSSION

In the above sections, we briefly outlined the dynamics of a simple continuum mechanics model in near-resonance and non-resonances cases. In the resonant case, where the frequency ration between the pendulum and rod was 1:2, we found the co-existence between periodic motion constrained to a surface, and high dimensional (in a KL measure) of chaotic motion. In the non-resonance case, we found low dimensional and high dimensional chaos co-existing in large regions of forcing amplitude. As a result of multiple attractors in the resonance cases, we defined and applied a measure of extreme parametric sensitivity, which ultimately obstructs asymptotic predictability in certain parameter regions of interest.

In addition to asymptotic attractors, there do exist temporary transient phenomena in this model as well as many other nonlinear systems, but might be long lived (Schwartz and Triandaf, 1996a), (Triandaf and Schwartz, 2000). Such long lived transients have all the characteristics of chaos, but they are not attracting. In Fig. 9, we see that for a truncated one mode model of Eqs. 1-2, a chaotic attractor exists at the right (open squares). However, for lower values of forcing, the attractor asymptotes to a period 2 attractor.

In many applications, it is desirable in the resonance cases, to keep the asymptotic periodic state from occurring. Solving this difficult control problem is indeed possible if one implements a topological procedure known as segmentation (Triandaf and Schwartz, 2000). That is, the goal of the control dynamics is to sustain chaos were there is none by using perturbations of the parameter α. Figure 9 illustrates the results of the procedure, where chaos (dots) is sustained over the whole region shown. In a measure theoretic sense based on the above definition of extreme parametric sensitivity, the technique increases ones predictability by eliminating the periodic attractors.

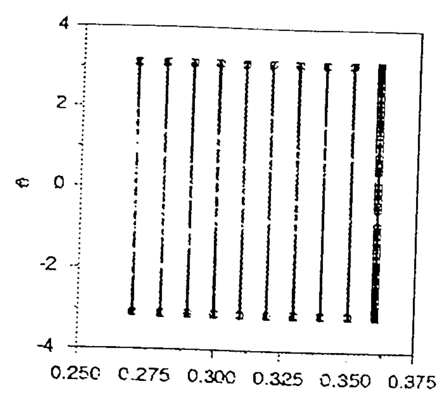

Figure 9. Transient and sustained chaos of a truncated model

The actual parameter fluctuations used to sustain chaos throughout the parameter region are shown as a function of time. Notice that as we move further away from the original chaotic attractor, the frequency of intervention increases.

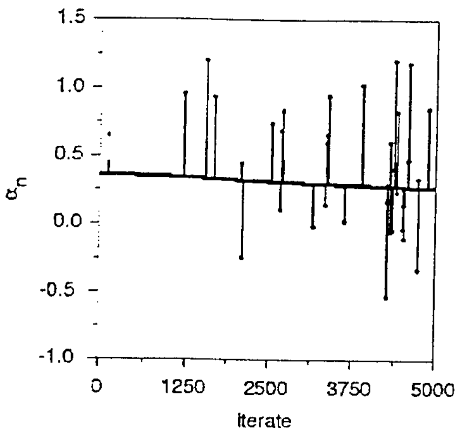

Figure 10.

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